Understanding nucleon structure using lattice simulations

Recent progress on three different structural observables

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Abstract. This review focuses on the discussion of three key results of nucleon structure calculations on the lattice. These three results are the quark contribution to the nucleon spin, J_q, the nucleon-Δ transition form factors, and the nucleon axial coupling, g_A. The importance for phenomenology and experiment is discussed and the requirements for future simulations are pointed out.

PACS. 12.38.Gc Lattice QCD calculations

1 Introduction

In recent years lattice gauge theory has become a mature and reliable way to investigate the structure of strong interactions. It provides a model-independent way to do calculations in QCD. However, contemporary lattice computations become extremely costly at quark masses corresponding to pion masses below 500 MeV. Nature, however, has chosen the pion mass to be only 140 MeV. The lightness of the pseudoscalar mesons is due to the mechanism of spontaneous chiral symmetry breaking. If, however, we can investigate only the regime of heavy quarks, where chiral symmetry is broken explicitly by the quark mass, we might not describe physics accurately at light-quark masses.

To address and overcome this challenge, three different procedures have been proposed and are actively pursued: i) pushing existing simulations with Wilson-type quarks down to smaller quark masses by relying on improved algorithms and faster computers [1], ii) using a hybrid action approach by using different formulations for sea and valence quarks [2], and iii) doing simulations using dynamical Ginsparg-Wilson formulations, such as domain-wall fermions [3] or overlap fermions [4]. The last approach is certainly the most challenging and demanding one since the entire parameter space has to be explored again. This applies also to heavy quarks, a regime in which Ginsparg-Wilson fermions are about 30 to 100 times more expensive than standard Wilson-type fermions.

The hybrid action ansatz is an excellent compromise between quark mass and performance, but suffers from conceptual problems. First of all, the hybrid theory breaks unitarity at finite lattice spacing. Thus, it cannot act as

an effective theory at finite lattice spacing, and the existence of the continuum limit is crucial. Furthermore, usually staggered-type quarks are being used for the sea with the square-root being taken of the determinant. It is not clear if the procedure of taking the square root commutes with taking the continuum limit, see, *e.g.*, [5] for a recent review. Finally, the matching of sea and valence quark masses is prescription dependent, and particular choices may give rise to additional possibly large $\mathcal{O}(a^2)$ artifacts [6].

In this review we focus on three observables with relevance to phenomenological and experimental applications. The first one is the quark contribution to the nucleon spin, J_q. The second one is the transition form factors of the nucleon-Δ transition. The third one is the nucleon axial coupling, g_A. The former two of these quantities have so far been understood qualitatively, but a precise matching between the light-quark regime and the lattice —possibly by chiral perturbation theory or an effective model of the strong interaction like [7]— still remains to be done. For the latter observable it has been shown that lattice data can in fact be consistent with experiment when fitting it using the leading logarithmic chiral perturbation theory expression. This achievement marks a milestone in the field of nucleon structure.

2 Quark contribution to nucleon spin

The quark contribution to the spin of the nucleon has been under intense scrutiny after the observation that only about $(20 \pm 15)\%$ of the nucleon spin arises from the quark spin [8]. Recently, it has been realized how the use of GPDs [9–11] provides the means to directly compute the quark contribution to the nucleon spin via the

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energy momentum tensor [10],

$$J_q = \lim_{t \to 0} \left(A_{20}^{u+d}(t) + B_{20}^{u+d}(t) \right) . \tag{1}$$

The virtuality t is given by $t \equiv (p' - p)^2$, where p' and p are the nucleon's incoming and outgoing momenta. The generalized form factors, $A_{20}^{u+d}(t)$ and $B_{20}^{u+d}(t)$, show up in the parameterization of the nucleon's energy-momentum tensor. For further details and the exact definition, consult [10]. The challenge is to understand which fraction of the nucleon spin, $J_N = 1/2$, arises from the quark spin, $1/2\Sigma_q$, the quark orbital angular momentum, L_q, and which fraction comes from gluon contributions, J_g:

$$J_N = 1/2 = J_q + J_g = 1/2\Sigma_q + L_q + J_g . \tag{2}$$

The value of Σ_q has been known before [12]. The new ingredient is the ability to directly calculate J_q, and thus also L_q. To this end, there is no experimental determination of that quantity. The first computation of J_q on the lattice has been done in [12]. This calculation only utilizes quenched Wilson fermions, but features a calculation of the disconnected contribution using noisy estimators. A later calculation [13] calculates all generalized form factors in the energy-momentum tensor separately and at the same time a publication [14] features full QCD and introduces an improved technology to extract form factors from matrix elements. Higher moments of GPDs have also been computed [15].

As of today, the understanding gained from the world of pions weighing 500 MeV and beyond is that the quark contribution to the nucleon spin is about 70%, all of which comes from the quark spin alone. The remaining 30% comes from the gluons. The quark orbital angular momentum is negligibly small due to a cancellation between the contributions of u- and d-quarks [2].

This result differs from the finding outlined above which indicates that this quantity can be expected to substantially depend on the pion mass. The cancellation of the orbital angular momentum for u- and d-quarks is an interesting qualitative feature. The insight that the nucleon in the heavy-pion world receives a larger fraction of its spin from quarks rather than gluons is compatible with expectations from the non-relativistic quark model, but the exact interpolation between the heavy-quark and the light-quark regime can give further insight into how the strong interaction operates.

The extrapolation to the chiral regime, however, has not yet been possible and hence a precise quantitative matching with nature has not yet been established. Although ref. [16] suggests a rather flat expression it is yet unclear whether the same straight line is to be used for the light-quark regime as the one fitting the simulations. In this situation it is inevitable to perform similar calculations at smaller pion masses before a matching between lattice and small-scale expansion schemes can be established and a definitive prediction from the lattice can be provided.

Further investigations from several groups are underway and all three different paths outlined in sect. 1 are taken to resolve this important question. We can conclude, however, that the technology and understanding of how to compute these matrix elements are available and can be deployed easily once sufficiently light pion masses are available.

3 N-Δ transition form factors

A key question is whether the baryon states of QCD are spherical or deformed. Although the nucleon is easily accessible in exclusive and inclusive scattering experiments, it cannot have a spectroscopic quadrupole moment since it has spin $J_N = 1/2$. The excited states with spin 3/2 and above can have a quadrupole moment, but these are not easily accessible in experiments. The only way to learn about deformations of the low-lying baryon spectrum is to consider transitions between the nucleon and the first-excited state, the $\Delta(1232)$-resonance. Experimentally, a flurry of activity has recently led to several important and exciting results [17].

The nucleon-Δ transition can be parameterized using three form factors —the dominant magnetic dipole form factor, $\mathcal{G}_{M1}$, the electric quadrupole, $\mathcal{G}_{E2}$, and the Coulomb quadrupole, $\mathcal{G}_{C2}$. Should the nucleon-Δ system be deformed, the latter two form factors will not vanish. Should the system be spherical, only the magnetic dipole form factor will be non-zero.

On the lattice, publications reporting the successful computation of these transition form factors are given in [18]. This set of calculations used unquenched Wilson fermions with pion masses beyond 600 MeV and quenched Wilson fermions with pion masses larger than 370 MeV. Later it has been attempted to apply these techniques also for hybrid actions [19], but to this end the statistical error bars on the quadrupole form factors turn out to be too large.

From these studies it has been clearly established, that the nucleon-Δ system is indeed deformed. The sign and the order of magnitude of the quadrupole form factors $\mathcal{G}_{E2}$ and $\mathcal{G}_{C2}$ were extracted successfully. The heavy-pion world in fact is similar to nature for these observables.

However, the extrapolation to the physical pion masses yields an inconsistency for $\mathcal{G}_{C2}$ at values below $Q^2 < 0.2\,\text{GeV}^2$. This discrepancy has been addressed recently in the framework of chiral perturbation theory [20]. It appears plausible that the discrepancy in fact arises from the inadequacy of a linear chiral extrapolation —it still remains to be seen if lattice data at smaller pion masses can indeed verify the pion mass dependence suggested in [20].

4 Nucleon axial coupling g$_A$

The investigation of the nucleon axial coupling has a long history on the lattice, see [21] for recent reviews. Several groups have performed investigations using a wide array of different lattice actions, spacings, volumes, and pion masses.
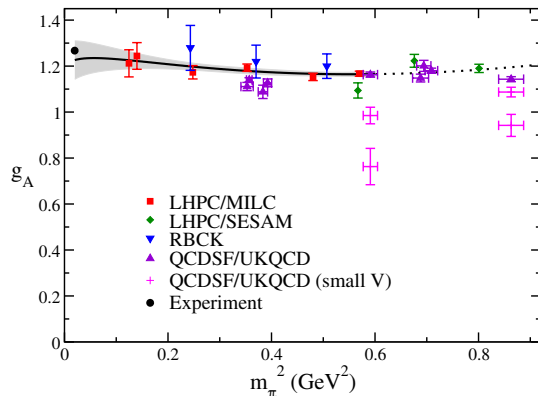
Fig. 1. Full QCD computations of the nucleon axial coupling, g_A. The line shows the fit of the leading logarithmic χPT expression to the hybrid lattice data from the LHPC Collaboration. Results from other groups are plotted, but not included in the fit. Figure taken from ref. [22], QCDSF data updated from ref. [23].

Recently, two independent papers, [22] and [23], have appeared showing how current lattice data can in fact be combined with chiral perturbation theory to arrive at the experimental value. Figure 1 shows the application of the leading logarithmic expression from χPT to the hybrid data computed by the LHPC Collaboration in [22]. The gray-shaded error band shows the error arising from statistical uncertainties only. The fit yields quantitative agreement with the experimental value.

However, the applicability of the leading-order chiral perturbation theory expression to the pion masses available has been questioned in [24]. The flat behavior at pion masses beyond 300 MeV is attributed to fine-tuning between different terms in the expansion. On the other hand, the expansion can still be consistent with experiment when applied to lattice calculations employing pion masses as large as 600 MeV [23]. It is perhaps fair to say that the exact range of applicability of χPT is under debate. Nonetheless, the striking agreement between the fit of lattice data and the experimental value mark an important milestone for the lattice treatment of nucleon structure.

5 Summary

We have given three examples of recent lattice calculations which are of great interest to both phenomenologists and experimentalists alike. The limiting factor of all these lattice results, however, is their limitation to rather large quark masses. Currently, the question of chiral extrapolations is under debate and the applicability depends strongly on the observable. While some groups successfully apply fits to pion masses as large as 600 MeV, other groups believe that pion masses lower than 300 MeV are essential. While the latter mass regime has not been reachable so far, we believe that the upcoming generation of lattice calculations will be able to settle the debate.

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